MMAT 5010 Linear Analysis (2023-24): Homework 10
Deadline: 20 Apr 2024

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let X be a Hilbert space and let $\{x_n : n = 1, 2...\}$ be an orthogonal subset of X. Show that the series $\sum_{k=1}^{\infty} x_k$ is convergent in X, that is $\lim_{N\to\infty} \sum_{k=1}^{N} x_k$ exists, if and only if $\sum_{k=1}^{\infty} \|x_k\|^2 < \infty$.
2. Let $(e_i)_{i\in I}$ and $(f_j)_{j\in I}$ be the orthonormal bases for the Hilbert spaces X and Y respectively. If for each $i \in I$, set $T(e_i) := f_i$, show that T can be extended to a unitary operator from X onto Y.

$***$ **End** $***$